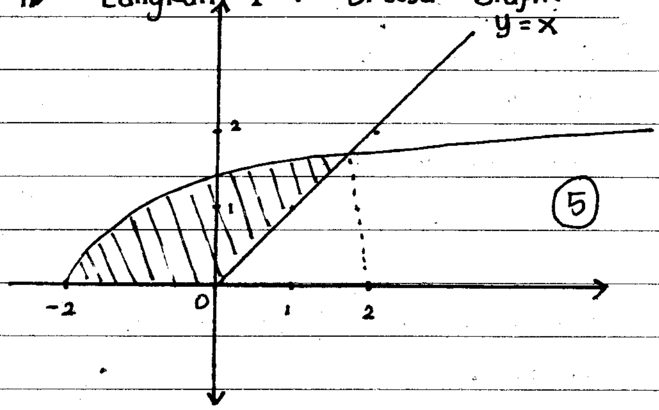
Kuis 2 - Kalkulus 2 (28)

1). $y = \sqrt{x+2}$; $y = x$; $y = 0$,
tentukan luasnya

▶ Langkah 1 : Sketsa Grafik



(5)

Titik potong :

$$y_1 = y_2$$
$$\sqrt{x+2} = x$$
$$x + 2 = x^2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2 \ \vee \ x = -1$$

(2)

▶ Langkah 2 : Merumuskan Integral

-> Terhadap x

$$L_1 = \int_{-2}^{0} \sqrt{x+2}\ dx$$

$$L_2 = \int_{0}^{2} \sqrt{x+2} - x\ dx$$

(8)

$$L_{total} = L_1 + L_2$$

atau

-> Terhadap y

$$L = \int_0^2 y - (y^2 - 2)\ dy$$

(8)

▶ Langkah 3 menyelesaikan Integral

-> Terhadap x

$$L_1 = \int_{-2}^{0} \sqrt{x+2}\ dx$$

misal : $p = x+2$ BA : $p = 2$

$dp = dx$ BB : $p = 0$

$$= \int_0^2 \sqrt{p}\ dp$$

$$= \frac{2}{3} p\sqrt{p} \Big|_0^2$$

$$= \frac{2}{3}(2\sqrt{2} - 0)$$

$$= \frac{4}{3}\sqrt{2}$$

$$L_2 = \int_0^2 \sqrt{x+2} - x\ dx$$

* $\int_0^2 \sqrt{x+2}\ dx$:

misal : $u = x+2$ BA : $u = 4$

$du = dx$ BB : $u = 2$

$$\int_2^4 \sqrt{u}\ du = \frac{2}{3} u\sqrt{u} \Big|_2^4$$

$$= \frac{2}{3}\left[4\sqrt{4} - 2\sqrt{2}\right]$$

$$= \frac{2}{3}\left[8 - 2\sqrt{2}\right]$$

$$= \frac{16}{3} - \frac{4}{3}\sqrt{2}$$

* $-\int_0^2 x\ dx = -\frac{1}{2}x^2 \Big|_0^2$

$$= -\frac{1}{2}\left[4\right]$$

$$= -2$$

$$L_2 = \frac{16}{3} - \frac{4}{3}\sqrt{2} - 2$$

$$= \frac{10}{3} - \frac{4}{3}\sqrt{2}$$

$$L_{tot} = L_1 + L_2 = \frac{4}{3}\sqrt{2} + \frac{10}{3} - \frac{4}{3}\sqrt{2}$$

$= \frac{10}{3}$ satuan luas (5)

-> Terhadap y

$$\int_0^2 y - (y^2 - 2)dy = \int_0^2 -y^2 + y + 2\ dy$$

$$= -\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y \Big|_0^2$$

$$= -\frac{8}{3} + 2 + 4$$

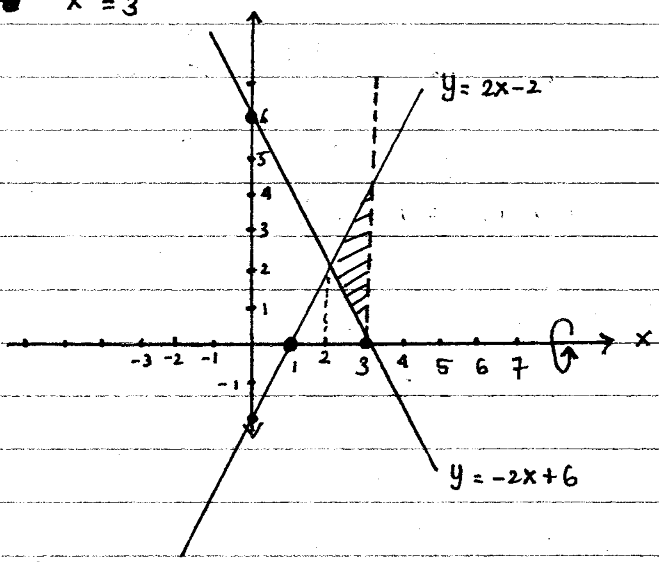
$$= 6 - \frac{8}{3}$$

$= \frac{10}{3}$ satuan luas. (5)

2). Volume benda padat : $y = 2x-2$, $y=-2x+6$, $x=3$ diputar pada sumbu-x

▶ Langkah 1 : Sketsa Grafik (5)

- $y = 2x - 2$
  - $y=0 \rightarrow x=1$ $(1,0)$
  - $x=0 \rightarrow y=-2$ $(0,-2)$
- $y = -2x+6$
  - $x=0 \rightarrow y=6$ $(0,6)$
  - $y=0 \rightarrow x=3$ $(3,0)$
- $x = 3$



▶ Merumuskan Volume

- Metode Cincin :

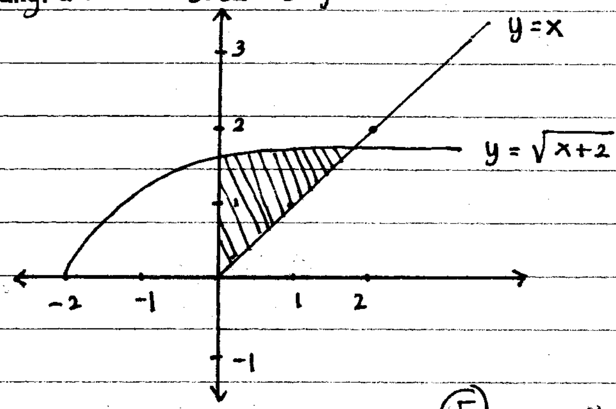
※ Menentukan titik potong

$$y_1 = y_2$$
$$2x-2 = -2x+6$$
$$4x = 8$$
$$x = 2$$

$$V = \int_2^3 \pi\left[(2x-2)^2 - (-2x+6)^2\right]dx \quad (10)$$
$$= \pi\int_2^3\left[4x^2-8x+4-(4x^2-24x+36)\right]dx$$
$$= \pi\int_2^3 4x^2-4x^2-8x+24x+4-36\ dx$$
$$= \pi\int_2^3 16x-32\ dx$$
$$= \pi\left[8x^2-32x\Big|_2^3\right]$$
$$= \pi\left[8[3]^2-32(3)-(8(2)^2-32(2))\right]$$
$$= \pi\left[8.9-32.3-8.4+32.2\right]$$
$$= \pi\left[8(9-4)+32[-3+2]\right]$$
$$= \pi\left[8.5+32(-1)\right]$$
$$= \pi\left[40-32\right]$$
$$= 8\pi \text{ satuan Volume} \quad (5)$$

3). Titik berat benda : $y=\sqrt{x+2}$, $y=x$, $y=0$ di Kuadran I.

▶ Langkah I : Sketsa Grafik



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※ Karena hanya Kuadran I

▶ Langkah 2 : Merumuskan $\bar{x}$ dan $\bar{y}$

$$\bar{x} = \frac{\int_0^2 x\left[\sqrt{x+2}-x\right]dx}{\int_0^2 \sqrt{x+2}-x\ dx} \quad (5)$$

$$\bar{y} = \frac{\frac{1}{2}\int_0^2\left[\sqrt{x+2}\right]^2 - x^2\ dx}{\int_0^2\sqrt{x+2}-x\ dx} \quad (5)$$

▶ Langkah 3 : Menghitung Integral

※ $\int_0^2 \sqrt{x+2} - x\ dx$

misal : $p = x+2 \rightarrow x = p-2$ BA : $p = 4$

$dp = dx$ BB : $p = 2$

$$= \int_2^4 \sqrt{p}-(p-2)\,dp = \int_2^4 \sqrt{p}-p+2\ dp$$
$$= \frac{2}{3}p\sqrt{p} - \frac{1}{2}p^2 + 2p\Big|_2^4$$

$$= \frac{2}{3}4\sqrt{4} - \frac{1}{2}(4)^2 + 2(4) - \left[\frac{2}{3}2\sqrt{2} - \frac{1}{2}.2^2 + 2.2\right]$$

$$= \frac{16}{3} - \frac{4}{3}\sqrt{2} - 8 + 2 + 8 - 4$$

$$= \frac{16}{3} - \frac{4}{3}\sqrt{2} - 2$$

$$= \frac{10}{3} - \frac{4}{3}\sqrt{2}$$

※ $\int_0^2 x\left[\sqrt{x+2} - x\right]dx = \int_0^2 x\sqrt{x+2} - x^2\,dx$

•> $\int_0^2 x\sqrt{x+2}\,dx$

misal : $u = x+2$ BA : $u = 2+2 = 4$

$du = dx$ BB : $u = 0+2 = 2$

$x = u-2$

$$= \int_2^4 (u-2)\sqrt{u}\,du = \int_2^4 u\sqrt{u} - 2\sqrt{u}\,du$$

$$= \frac{2}{5}u^2\sqrt{u} - 2.\frac{2}{3}u\sqrt{u}\Big]_2^4$$

$$= \frac{2}{5}u^2\sqrt{u} - \frac{4}{3}u\sqrt{u}\Big|_2^4$$

$$= \frac{2}{5}\cdot 4^2\sqrt{4} - \frac{4}{3}4\sqrt{4} - \left[\frac{2}{5}2^2\sqrt{2} - \frac{4}{3}2\sqrt{2}\right]$$

$$= \frac{64}{5} - \frac{32}{3} - \left[\frac{8}{5}\sqrt{2} - \frac{8}{3}\sqrt{2}\right]$$

$$= \frac{192 - 160 - 24\sqrt{2} + 40\sqrt{2}}{15}$$

$$= \frac{32}{15} + \frac{16}{15}\sqrt{2}$$

•> $-\int_0^2 x^2\,dx = -\frac{1}{3}x^3\Big|_0^2$

$$= -\frac{8}{3}$$

$$\int_0^2 x\left[\sqrt{x+2} - x\right]dx = \frac{32}{15} + \frac{16}{15}\sqrt{2} - \frac{8}{3}$$

$$= \frac{32-40}{15} + \frac{16}{15}\sqrt{2}$$

$$= \frac{16}{15}\sqrt{2} - \frac{8}{15}$$

※ $\frac{1}{2}\int_0^2 \left[\sqrt{x+2}\right]^2 - x^2\,dx = \frac{1}{2}\int_0^2 x+2-x^2\,dx$

$$= \frac{1}{2}\left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\Big|_0^2\right]$$

$$= \frac{1}{2}\left[-\frac{8}{3} + 2 + 4\right]$$

$$= \frac{1}{2}\left[6 - \frac{8}{3}\right]$$

$$= \frac{1}{2}\left[\frac{10}{3}\right]$$

$$= \frac{5}{3}$$

▶ Langkah 4 : Substitusi hasil

$$\bar{x} = \frac{\frac{4\cancel{8}\left[-1+2\sqrt{2}\right]}{\cancel{15}\,5}}{\frac{\cancel{2}\left[5-2\sqrt{2}\right]}{\cancel{3}}}$$

$$= \frac{4}{5}\,\frac{(2\sqrt{2}-1)}{(5-2\sqrt{2})}\cdot\frac{(5+2\sqrt{2})}{(5+2\sqrt{2})}$$

$$= \frac{4}{5}\,\frac{(10\sqrt{2} + 8 - 5 - 2\sqrt{2})}{25-8}$$

$$= \frac{4}{5}\,\frac{(8\sqrt{2}+3)}{17}$$

$$= \frac{32\sqrt{2} + 12}{85}$$

$$\bar{y} = \frac{5/\cancel{3}}{\frac{2\left[5-2\sqrt{2}\right]}{\cancel{3}}}$$

$$= \frac{5}{2}\cdot\frac{1}{5-2\sqrt{2}}\cdot\frac{(5+2\sqrt{2})}{(5+2\sqrt{2})}$$

$$= \frac{5}{2}\cdot\frac{5+2\sqrt{2}}{25-8}$$

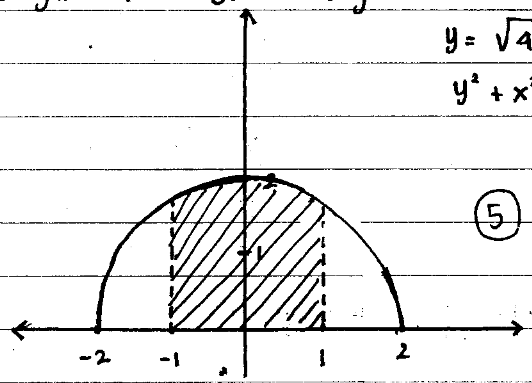
$$= \frac{25+10\sqrt{2}}{34}$$

Jawab : $(\bar{x},\bar{y}) = \left(\frac{32\sqrt{2}+12}{85}, \frac{25+10\sqrt{2}}{34}\right)$ //

4). Sketsa busur yang dibatasi $y = \sqrt{4-x^2}$, $y=0$, $-1 \le x \le 1$.

Dapatkan luas kulit dengan sumbu putar sumbu-x dengan dalil Guldin II. Jika titik berat di $Z(0, \frac{6}{\pi})$

▶ Langkah 1 : Sketsa Grafik



$y = \sqrt{4-x^2}$

$y^2 + x^2 = 4$

5

▶ Langkah 2 : Tentukan panjang Kurva

$$S = \int_{-1}^{1} \sqrt{1 + [f'(x)]^2}\, dx$$

$$\Rightarrow f'(x) = \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{4-x^2}} \cdot -\cancel{2}x$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$\Rightarrow [f'(x)]^2 = \frac{x^2}{4-x^2}$$

$$S = \int_{-1}^{1} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx \qquad 5$$

$$= \int_{-1}^{1} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$$

$$= \int_{-1}^{1} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= \int_{-1}^{1} \frac{2}{\sqrt{4-x^2}}\, dx$$

$$= 2 \cdot \sin^{-1}\frac{x}{2} \Big|_{-1}^{1}$$

$$= 2\left[\sin^{-1}\frac{1}{2} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$

$$= 2\left[\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right] \qquad 3$$

$$= 2 \cdot \frac{2\pi}{6} = \frac{2\pi}{3} \text{ Satuan panjang}$$

▶ Langkah 3 : Menentukan jarak sumbu putar ke titik berat

$$d = \frac{6}{\pi} \qquad 2$$

▶ Langkah 4 : Menentukan Luas kulit dengan Dalil Guldin II

$$K = 2\pi \cdot d \cdot S$$

$$= 2\pi \cdot \frac{\cancel{6}^{2}}{\cancel{\pi}} \cdot \frac{2\cancel{\pi}}{\cancel{3}}$$

$$= 8\pi \text{ Satuan luas} \qquad 5$$

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